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MAGNETOPHORETIC POTENTIAL OF A CHAIN OF FERROMAGNETIC BALLS IN A HOMOGENEOUS FIELD

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The class of filtering magnetic structures formed by an ordered set of ferromagnetic microballs is considered. We develop an approach to the analysis of their magnetophoretic properties based on the dipole approach and the concept of a magnetophoretic potential averaged along the direction of motion of the suspension of separated particles. We have investigated the magnetophoretic properties of longitudinally and transversely magnetized linear chains of balls. Experiments aimed at estimating the accuracy of the dipole approximation have been performed.

The magnetic separation method, which has been used in the mining-enriching industry for more than a hundred years, since the mid-70s of the last century has been attracting attention in many other fields of activities, including water purification, gas cleaning, cleaning of clays, chemical technologies, medicine, and biology (see [1–7]). The widespread interest in magnetic separation is due to the realization that, by means of a magnetic field combining a high intensity with a small-scale homogeneity, one can separate from a gas or liquid flow even quite small low-magnetic objects, including particles of cell suspensions. The direction known as "high-gradient magnetic separation" (HGMS) has been formed. In practice, high-gradient magnetic filters are made by applying a strong magnetic field to a volume in which small ferromagnetic bodies are distributed. The smaller the separated particles and the weaker their magnetic properties, the stronger should be the external field and the smaller the size of the ferromagnetic head elements. If ferromagnetic particles of iron oxides from a thermoelectric plant condensate are separated in the charge of metal balls of sizes of a few millimeters [4], then the scale of the field inhomogeneity needed to separate cell suspensions is of the order of 100 μm. A charge of such small ferromagnetic granules is rather impermeable. Certain advances have been made with the use of bundles or a sparse packing of fine ferromagnetic wire [2, 5]. In general, however, the question of the optimum structure of fine magnetic filters is still open and is of great theoretical and practical interest. A separate class of filtering magnetic structures can be formed by structures formed somehow by an ordered set of identical ferromagnetic balls. In this work, we develop an approach to their analysis. It is based on the dipole approach for describing the field distribution and the concept of a magnetophoretic potential averaged along the direction of motion of the suspension of separated particles. As an example, we have investigated the magnetophoretic potential of an infinite linear chain of magnetized balls oriented in the direction of motion of the suspension. The results of the experiments performed with the aim of checking the accuracy of the dipole approximation are also presented.

General Consideration. We investigate a filtering structure representing a set of identical spherical magnetic balls magnetized by a homogeneous field $\mathbf{H}_0 = H_0\mathbf{e}$. The structure geometry is fully characterized by a set of radius vectors $\mathbf{R}_\alpha$ drawn into the particle centers. We exclude the influence of the balls on the magnetization of one another and assume that all of them are magnetized homogeneously. In this case, the magnetic field created by a ball α at any outer, with respect to it, point A (with a radius vector $\mathbf{R}_A$) is the field of a point magnetic dipole situated at point $\mathbf{R}_\alpha$ and having the quantity $m = (4/3)\pi a^3 M$ and the direction $\mathbf{e}$:

$$\mathbf{H}'_\alpha(\mathrm{A}) = -\frac{m}{R^3_{\alpha\mathrm{A}}}\left[\mathbf{e} - 3\left(\mathbf{e}\boldsymbol{\rho}_{\alpha\mathrm{A}}\right)\boldsymbol{\rho}_{\alpha\mathrm{A}}\right]. \tag{1}$$

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Here $\rho_{\alpha A} = (\mathbf{R}_A - \mathbf{R}_\alpha)/R_{\alpha A}$; $R_{\alpha A} = |\mathbf{R}_A - \mathbf{R}_\alpha|$. Using the ball radius as a spatial scale and introducing the designation $r_{\alpha A} = R_{\alpha A}/a$, we rewrite (1) in the form

$$\mathbf{H}'_\alpha(A) = -\frac{4\pi M}{3r^3_{\alpha A}}\,[\mathbf{e} - 3\,(\mathbf{e}\rho_{\alpha A})\,\rho_{\alpha A}]\,. \tag{2}$$

The field strength of the particle aggregate at point A under the assumptions made is obtained by summing fields (2) of separate particles. Let us write it in the form

$$\mathbf{H}'(A) = \frac{8\pi M}{3}\,\mathbf{h}(A)\,,\quad \mathbf{h}(A) = -\frac{1}{2}\sum_\alpha \frac{1}{r^3_{\alpha A}}\,[\mathbf{e} - 3\,(\mathbf{e}\rho_{\alpha A})\,\rho_{\alpha A}]\,. \tag{3}$$

The total field strength also includes the external field strength: $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}'$.

Let the system being separated be an aggregate of weakly magnetic (dia- or paramagnetic) microparticles suspended in a liquid or gaseous medium. We assume that the size of separated particles is small as compared to the diameter of the ferromagnetic balls. Under these conditions, it may be assumed that the magnetic field of the structure inside the separated microparticles is homogeneous and the demagnetizing field of the microparticles can be neglected. In this case, the expression for the magnetic force acting on an individual microparticle is of the form

$$\mathbf{F} = \frac{1}{2}\,\Delta\chi v \nabla H^2\,, \tag{4}$$

where $\Delta\chi = \chi - \chi_0$. Using the relation $H^2 = H_0^2 + H'^2 + 2\mathbf{H}_0\mathbf{H}'$ and taking into account the external field homogeneity, we reduce (4) to the form

$$\mathbf{F} = \frac{1}{2}\,\Delta\chi v \left(\frac{8\pi M}{3}\right)^2 \nabla\,[h^2 + P\mathbf{eh}]\,, \tag{5}$$

where the complex $P = 3H_0/(4\pi M)$ is the ratio of the external field strength to the demagnetizing field strength of the magnetic sphere.

Relation (5) permits introduction of a magnetophoretic potential Φ of the filtering structure:

$$\mathbf{F} = -\nabla\Phi\,,\quad \Phi = -\frac{1}{2}\,\Delta\chi v \left(\frac{8\pi M}{3}\right)^2 [h^2 + P\mathbf{eh}]\,. \tag{6}$$

Let us introduce a dimensionless magnetophoretic potential, using, as a scale, the potential value of the sphere magnetized to saturation ($M = M_s$) at its frontal point ($\rho_{\alpha A} = \mathbf{e}$) with a hypothetically switched-off field:

$$\Phi^* = \frac{1}{2}\,\Delta\chi v \left(\frac{8\pi M_s}{3}\right)^2 .$$

We have

$$\varphi \equiv \Phi/\Phi^* = -M_r^2 h^2 - PM_r\mathbf{eh}\,,\quad M_r = M/M_s\,. \tag{7}$$

In the state of magnetic saturation the magnetophoretic potential takes on the form

$$\varphi = -h^2 - P_s\mathbf{eh}\,,\quad (P_s = 3H_0/4\pi M_s > 1)\,. \tag{8}$$

Saturation is attained in an external field exceeding the demagnetizing field of the sphere magnetized to saturation ($H_0 > 4\pi M_s/3$). If the material of the ferromagnetic balls is pure iron (M_s = 1700 G), then saturation occurs in a field of strength above 7 kOe. Note that if the magnetic structure of the filter is magnetized to saturation, the dipole approximation used by us is exact. In the linear portion of magnetization, the sphere magnetization with regard for the demagnetizing field is given by the relation

$$M = \frac{\chi_f H_0}{1 + (4\pi\chi_f/3)}\,.$$

Since $\chi_f > 1$, from this $M = 3H_0/4\pi$ follows and $P = 1$. In further considerations, we use, as the basis, the magnetic saturation limit, i.e., relation (8), which we give in the form

$$\varphi = \varphi_1 + P_s\,\varphi_2\,, \quad \varphi_1 = -\,h^2\,, \quad \varphi_2 = -\,\mathbf{eh}\,. \tag{9}$$

Note that the value of Φ^* used to dedimensionalize the potential can be positive or negative depending on the sign of $\Delta\chi$. The latter can change depending on the magnetic properties of the medium. However, for any practical purposes it is convenient to consider $\Delta\chi$ as an effective susceptibility of particles suspended in a nonmagnetic medium and speak of paramagnetic ($\Delta\chi > 0$) and diamagnetic ($\Delta\chi < 0$) particles. Then the magnetophoretic potential dedimensionalized by the adopted technique pertains to paramagnetic particles and that taken with the opposite sign pertains to diamagnetic particles. Paramagnetic particles move in the direction of the minimum of the potential φ and diamagnetic ones — in the direction of its maximum.

Note that the first term in (9) is independent of the external field strength and the second term is proportional to the field strength. In the dimensional form, it is of the form

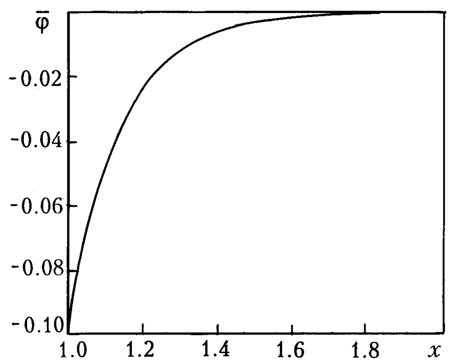
$$P_s\,\varphi_2\Phi^* = -\,(1/3)\;8\pi\nu\Delta\chi M_s H_0\,(\mathbf{eh})\,.$$

As follows from this expression, the magnetophoretic potential, apart from the volume and susceptibility of separated particles, the saturation magnetization of ferromagnetic spheres, and the value of the external field, is determined by the direction of the latter and the structure of the eigenfield of the system of spheres. Obviously, the whole set of minima and maxima of the magnetophoretic potential determined by relations (9) and (3) is at dipole localization points, so that both diamagnetic and paramagnetic particles under the action of the magnetophoretic force stream to the centers of magnetic balls and deposit on different parts of their surface. In flowing schemes, which are of particular practical interest, the probability of capture of a particle as it is moving in the flow near a separate ball is finite and decreases with increasing flow rate and distance from the ball to the streamline. If the balls are randomly distributed, then the change of the particle path under the action of one ball, which has not led to the capture of a particle, on average does not influence the probability of its capture by the balls situated lower in the stream. In this case, the cleaning depth can be increased by using extensive factors: by decreasing the flow rate and increasing the concentration of balls and the length of the head. Another approach is more promising for the theory and practice of high-gradient magnetic separation. It is based on the idea of forming an ordered structure of magnetic balls that would provide the effect of accumulation of magnetophoretic displacement of a separated particle towards the deposition centers as the particle is moving along the head from one ball to another. An example of such a structure is a rectilinear chain of balls directed along the suspension flow.

Magnetophoretic Potential of a Linear Chain. We define the chain direction by the unit vector **n**. Let us place the origin of the system of coordinates at the center of the particle with number 0. The position of the particle with number α is given by the radius vector $\mathbf{R}_\alpha = 2\alpha\mathbf{n}$. The number α assumes integer values from $-m$ to m. The number of particles in the chain $n = 2m + 1$. Assuming m to be fairly large and considering the region in the vicinity of the chain axis and away from its ends, we model an infinite chain. Let us investigate the following two cases: the field is directed along the chain and transverse to it.

The chain is magnetized along its axis. By virtue of the problem symmetry, it is enough to consider the potential in any one plane containing a chain. Let it be the xz plane. The z axis is directed along the chain ($\mathbf{e} = \mathbf{k}$). Relations (9) and (3) for the magnetophoretic potential at point A (x, 0, z) take on the form

$$\varphi = \varphi_1 + P_s\,\varphi_2\,, \quad \varphi_1 = -\,(h_x^2 + h_z^2)\,, \quad \varphi_2 = -\,h_z\,,$$



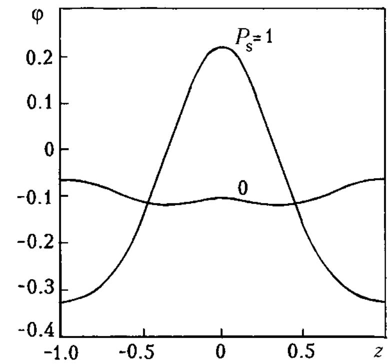


Fig. 1. Mean magnetophoretic potential of the longitudinally magnetized chain of balls versus the distance to its axis.

Fig. 2. Change in the local magnetophoretic potential of the transversely magnetized chain of balls along its axis.

$$\mathbf{h}\,(x,0,z)=\frac{3x\mathbf{i}}{2}\sum_{\alpha}\frac{z-2\alpha}{r_{\alpha A}^{5}}-\frac{\mathbf{k}}{2}\sum_{\alpha}\frac{1}{r_{\alpha A}^{3}}\left[1-3\,\frac{(z-2\alpha)^{2}}{r_{\alpha A}^{2}}\right],\quad r_{\alpha A}=[(z-2\alpha)^{2}+x^{2}]^{1/2}\,. \tag{10}$$

As the suspension of low-magnetic particles moves along an infinite magnetized chain, they are subjected to the action of a phoretic force having components along the chain ($\mathbf{f}_{\|}$) and transverse to it ($\mathbf{f}_{\perp}$). For fixed x, the values of the longitudinal and transverse components of the force periodically change, and the transverse force thereby has a nonzero mean value $\bar{f}_{\perp}(x)$ and the mean value of the longitudinal force is equal to zero. Under the action of the steady component of the transverse force, particles can continuously move towards the chain, where they will accumulate in a periodic system of potential wells on the surface of magnetic balls. Under the condition that the suspension velocity exceeds the velocity of phoretic displacement, the value of the magnetophoretic effect can be characterized by the transverse force averaged over the chain period in the case of motion along a straight line x = const:

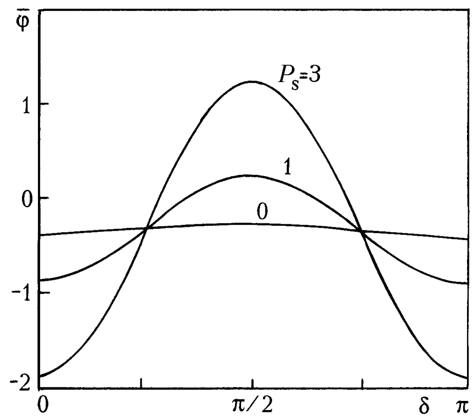
$$\bar{f}_{\perp}\,(x)=-\int_{0}^{1}\frac{\partial\varphi\,(x,z)}{\partial x}\,dz=-\frac{\partial\bar{\varphi}}{\partial x}\,,\quad \bar{\varphi}\,(x)=\int_{0}^{1}\varphi\,(x,z)\,dz\quad (x\geq 1)\,. \tag{11}$$

As the calculations show, the dependence of the mean potential on the distance to the chain axis (Fig. 1) is the same for all values of P. Consequently, the increase in the field strength as magnetic saturation is reached has no consequences for the mean potential. This is explained by the fact that the component $\bar{\varphi}_2$ of the mean potential is equal to zero. According to Fig. 1, the chain magnetized along its axis on average attracts paramagnetic particles and repulses diamagnetic ones. Confinement of deposited particles in the flow should be provided by a periodic (with a period equal to the distance between balls) system of extrema of the magnetophoretic potential along the magnetized chain. The dependence $\varphi(z)$ for the values x = 1, P_s = 0 and 1 is given in Fig. 2. As we can see, when the external field is off (P_s = 0), which corresponds to a chain of permanent magnets, the change in the potential along the chain is relatively weakly pronounced. On the contrary, in the case of magnetosoft balls (P_s = 1), the potential has deep valleys in the contact zones of balls. With increasing P_s the potential inhomogeneity along the chain increases.

The chain is magnetized transverse to its axis. Let us introduce a Cartesian system of coordinates with the z axis along the chain axis and the x axis along the magnetizing field. We also use cylindrical coordinates, the distance from the chain axis $r=(x^2+y^2)^{1/2}$, and the angle δ from the x axis, and write the relations for the potential in the form

$$\varphi_1=-\,(h_x^2+h_y^2+h_z^2)\,,\quad \varphi_2=-h_x\,,\quad \mathbf{r}_{\alpha A}=r\cos\delta\,\mathbf{i}+r\sin\delta\,\mathbf{j}+(z-2\alpha)\,\mathbf{k}\,,\quad r_{\alpha A}=(z-2\alpha)^2+r^2\,,$$

$$\mathbf{h}\,(r,\delta,z)=-\frac{\mathbf{i}}{2}\sum_{\alpha}\frac{1}{r_{\alpha A}^{3}}\left(1-\frac{3r^{2}\cos^{2}\delta}{r_{\alpha A}^{2}}\right)+\mathbf{j}\,\frac{3r^{2}\sin\delta\cos\delta}{2}\sum_{\alpha}\frac{1}{r_{\alpha A}^{5}}+\mathbf{k}\,\frac{3r\cos\delta}{2}\sum_{\alpha}\frac{z-2\alpha}{r_{\alpha A}^{5}}\,. \tag{12}$$



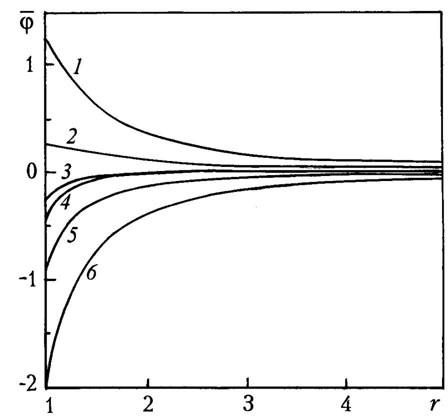


Fig. 3. Angular dependence of the mean magnetophoretic potential at distance $r = 1$ to the axis of the transversely magnetized chain of balls at various values of P_s.

Fig. 4. Mean magnetophoretic potential of the transversely magnetized chain of balls versus the distance to its axis in planes $\delta = 0$ (4, 5, 6) and $\delta = \pi/2$ (1, 2, 3) in magnetic fields of $P_s = 0$ (3, 4), $P_s = 1$ (2, 4), and $P_s = 3$ (1, 6).

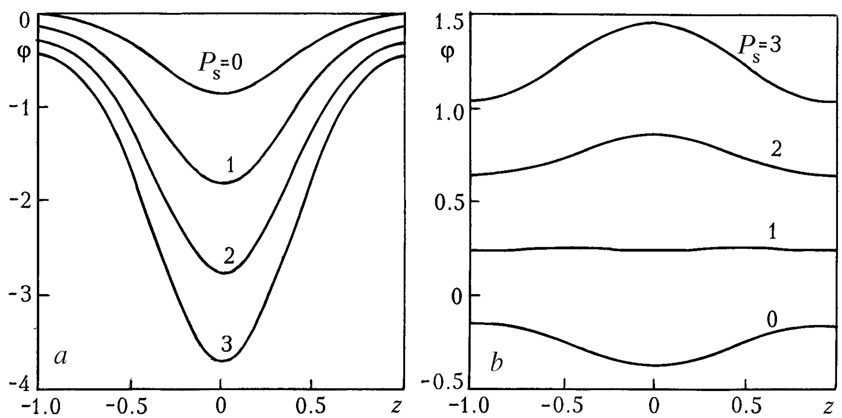


Fig. 5. Change in the local magnetophoretic potential along the transversely magnetized chain of balls in the plane $\delta = 0$ (a) and $\delta = \pi/2$ (b) at various values of P_s.

The dependence of the mean potential on the angle δ for $r = 1$ and several values of P is given in Fig. 3. We will call the plane (x, z) in which the minimum of the mean magnetophoretic potential lies the minimum plane and the plane (y, z) — the maximum plane. The dependence of the mean potential on the distance to the chain axis in the minimum and maximum planes is depicted in Fig. 4 for various P_s. Note that in the absence of the external field ($P_s = 0$) the magnetic chain (chain of permanent magnets) in both planes attracts paramagnetic particles and repulses diamagnetic ones. At the same time, the magnetized chain of magnetosoft particles ($P_s \geq 1$) is attractive for paramagnetic particles in the minimum plane and for diamagnetic particles — in the maximum plane. The local potential distribution along the chain for $r = 1$ and various P_s is given in Fig. 5a in the plane $\delta = 0$ and in Fig. 5b — in the plane $\delta = \pi/2$. As follows from the data presented, in the plane $\delta = 0$ (the plane of deposition of paramagnetic particles), the potential inhomogeneity along the chain is pronounced much more strongly than in the plane $\delta = \pi/2$ (the plane of deposition of diamagnetic particles).

Experimental Study of the Magnetic Field of the Chain of Spheres. *Experimental procedure.* The experimental study has been made in order to estimate the degree of fitness of the dipole approximation used to reality. The source of the magnetic field is an electromagnet with square pole pieces (square side of 90 mm, pole saturation of 32 mm). The magnetic chain was formed from bearing balls of radius $a = 3$ mm. The balls are packed closely to one

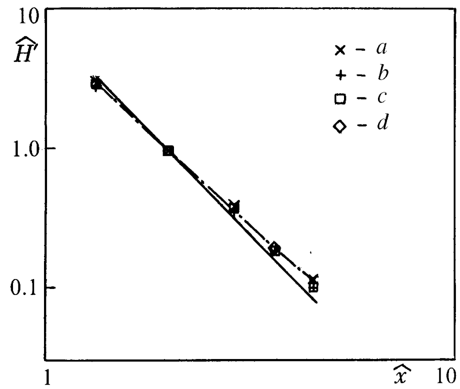


Fig. 6. Measured (dots) and calculated (curves) dependences of the field of the ball on the distance to the ball axis. Measurements were taken in fields of 1 (a), 3 (b), 6 (c), and 8 (d) kOe.

another in a race made in the form of a linear chain of holes cut in a nonmagnetic plate. The plate with a chain oriented transverse to the field is placed between the electromagnet poles. The field strength H is measured on the line of induction passing through the center of a particle located at the center of the chain. This line lies on the x-axis of the coordinate system taken higher in the theoretical investigation of the magnetophoretic potential of the transversely magnetized chain, and the magnetic field strength thereby has only an x-component because of the symmetry of the system under investigation. Simultaneously with H the field strength H_0 in the interpole space away from the particles is measured. Measurements are taken by two identical film Hall probes, one of which measures the value of H_0 and the other — H. The probes are connected to a differential voltmeter and permit registration of the difference $H' = H - H_0$, i.e., the field strength of the chain. The magnetizing field H_0 is varied between 0 and 8 kOe, the distance x from the chain axis to the probes takes a number of values from 4 to 5 mm, and the number of balls in the chain n = 1, 3, 5, 7.

Analysis of the experimental results. First consider the case of one particle. As is known, in a homogeneous external field an isotropic spherical body is magnetized homogeneously, and the eigenfield of such a body outside it is exactly the field of the magnetic dipole positioned at the sphere center and having a value equal to the value of the magnetic moment of the body. The dependence of the field strength of a solitary sphere on the distance to its center on the x axis is of the form

$$H'(x, 1) = \frac{8\pi M}{3}\left(\frac{a}{x}\right)^3 . \tag{13}$$

According to (13), at any strength of the magnetizing field the value of H' at an arbitrary point x assigned to the value of H' at some fixed point x^* should obey the universal dependence

$$\hat{H}'(\hat{x}, 1) = \hat{x}^{-3}, \quad \hat{H}'(\hat{x}, 1) = \frac{H'(x, 1)}{H'(x^*, 1)}, \quad \hat{x} = \frac{x}{x^*} . \tag{14}$$

Dependence (14) is given in Fig. 6 (solid line) together with the experimental data obtained for several values of the magnetizing field strength. As we see, the measured value of the ball's field strength decreases with distance somewhat slower compared to dependence (14). The observed discrepancy may be due to two reasons connected with the experimental conditions. First, this is the action of the specimen being investigated on the measuring system, in our case — the influence of the ball's magnetic field on the magnetic circuit magnetization. The dominating contribution of this factor can be taken into account by the reflection method, staying within the framework of the dipole approximation. With such an approach to the field of the ball in the interpolar space it is necessary to add the field of its two images lying on the x axis symmetrically about the flat surfaces of either pole of the electromagnet. The result of the calculation by such a scheme is represented in Fig. 6 by a dash-dot line. As is seen, accounting for the influence of the specimen on the system brings the theory closer to the experiment at all distances from the ball except for the

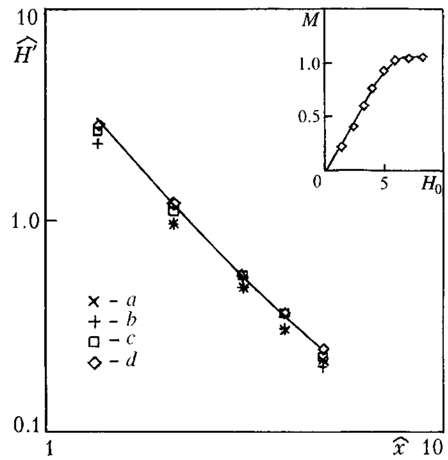


Fig. 7. Calculated (curve) and measured (in fields of different values) (for designations of points, see Fig. 6) dependences of the eigenfield of the chain of three balls on the distance to its axis. M, T; H_0, kOe.

region adjacent to its surface, where the changed strength values remain lower than the calculated ones. This difference can be explained by the finite sizes of the probe, which in the theory is taken to be a point probe. Let us estimate the error associated with this factor. Assuming for simplicity that the probe has the shape of a circle of area S, we find that the ball field strength H'_{av} (average over the probe surface) measured by it is related to the strength H' on the x axis by the relation

$$H'_{av} = H'\left(1 - \frac{3S}{2\pi x^2} + O\left(\frac{S^2}{x^4}\right)\right). \tag{15}$$

In our case, $S = 1$ mm^2, $x = 4$ mm (the change closest to the ball surface), and the relative systematic error, according to (15), is –3%. Actually, the values of the ball field strength measured at a corresponding point in different external fields are –3 to –8% smaller than the theoretical values. The additional (random) error on the low side of the measured value can arise because of the error made in positioning the probe on the measurement line.

The measurement data for the magnetic field strength of the chain of three balls are presented in Fig. 7. The same figure shows the theoretical dependences obtained in the dipole approximation both with and without taking into account the field of magnetic images of the chain in the electromagnet poles. The investigation of five and seven particles gave a similar result in comparing the calculated and measured dependences. The data obtained enable us to draw the following conclusions. First of all, note the expected fact that with magnetic saturation of particles the exactness of the dipole approximation increases. Indeed, while in the magnetizing fields of 1 and 3 kOe the measured values of the chain's eigenfield strength lie much lower than the calculated curve; in the fields of 6 and 8 kOe they practically coincide with it. In this connection, it is appropriate to give the curve of the dependence of the magnetization of an individual ball on the strength of the external magnetizing field. This dependence can be easily reconstructed with the aid of relation (13) by the measurement data for the field of an individual particle in various external fields at a fixed distance x from the particle center. It is given in the insert in Fig. 7 and shows that in the 6-kOe field the balls are really close to the magnetic saturation. Away from the magnetic saturation of balls the chain field turns out to be somewhat smaller than the calculated value. However, it should be noted that the difference between them has a purely multiplicative character and can be eliminated by choosing an effective value of particle magnetization in the chain that is somewhat smaller compared to the magnetization of an individual particle.

NOTATION

a, ferromagnetic ball radius; $\mathbf{H}_0$, external field strength; $\mathbf{H}'$, strength of the eigenfield of the magnetic structure; M, ball magnetization; v and χ, volume and magnetic susceptibility of the separated particle; χ_0, magnetic susceptibility of the carrying medium; χ_f, susceptibility of the sphere material. Subscripts: s, saturation; f, ferromagnetic; α, ball number; av, average.

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